Question			Answer	Marks	Guidance	
1	(i)		$\begin{pmatrix}-1\\4\\1\end{pmatrix}\times\overrightarrow{BC}=\begin{pmatrix}-1\\4\\1\end{pmatrix}\times\begin{pmatrix}-6\\18\\3\end{pmatrix}=\begin{pmatrix}-6\\-3\\6\end{pmatrix}$ $\left[=3\begin{pmatrix}-2\\-1\\2\end{pmatrix}\right]$	M1* A1	Vector product of directions	*Intention sufficient*
			Shortest distance is $\dfrac{\overrightarrow{AB}\mathbf{.}\mathbf{d}}{\lvert\mathbf{d}\rvert}=\dfrac{\begin{pmatrix}8\\-2\\-13\end{pmatrix}\mathbf{.}\begin{pmatrix}-2\\-1\\2\end{pmatrix}}{\sqrt{2^2+1^2+2^2}}$	M1* M1	Appropriate scalar product; Evaluation of $\lvert\mathbf{d}\rvert$	*Dep* *; *Dep* **
			Shortest distance is $\dfrac{40}{3}$	A1 **[5]**		
		OR	$\left[\begin{pmatrix}11-6\lambda\\18\lambda\\-3+3\lambda\end{pmatrix}-\begin{pmatrix}3-\mu\\2+4\mu\\10+\mu\end{pmatrix}\right]\mathbf{.}\begin{pmatrix}-1\\4\\1\end{pmatrix}=0$ and $\begin{pmatrix}8-6\lambda+\mu\\-2+18\lambda-4\mu\\-13+3\lambda-\mu\end{pmatrix}\mathbf{.}\begin{pmatrix}-6\\18\\3\end{pmatrix}=0$		M1* Two appropriate scalar products	
			$81\lambda-18\mu=29,\ \ 123\lambda-27\mu=41$		A1 Two correct equations	
			$\lambda=-\dfrac{5}{3},\ \mu=-\dfrac{82}{9},\ \overrightarrow{XY}=\begin{pmatrix}80/9\\40/9\\-80/9\end{pmatrix}$		M1* Obtaining $\overrightarrow{XY}$	*Dep* *
			Shortest distance is $\sqrt{\left(\dfrac{80}{9}\right)^2+\left(\dfrac{40}{9}\right)^2+\left(\dfrac{80}{9}\right)^2}$		M1	*Dep* **
			Shortest distance is $\dfrac{40}{3}$		A1	

Question			Answer	Marks	Guidance	
1	(ii)		$\overrightarrow{AB}\times\overrightarrow{BC}=\begin{pmatrix}8\\-2\\-13\end{pmatrix}\times\begin{pmatrix}-6\\18\\3\end{pmatrix}=\begin{pmatrix}228\\54\\132\end{pmatrix}$	M1* A2	Appropriate vector product Give A1 if one error	
			$\left\|\overrightarrow{AB}\times\overrightarrow{BC}\right\|=\sqrt{228^2+54^2+132^2}$ $\left\|\overrightarrow{BC}\right\|=\sqrt{6^2+18^2+3^2}$	M1*		*Dep **
			Shortest distance is $\dfrac{\left\|\overrightarrow{AB}\times\overrightarrow{BC}\right\|}{\left\|\overrightarrow{BC}\right\|}=\sqrt{\dfrac{72324}{369}}$	M1		*Dep ***
			Shortest distance is 14	A1 **[6]**		*Sign error in vector product can earn M1A1M1M1A1*
		OR	$\left[\begin{pmatrix}11-6\lambda\\18\lambda\\-3+3\lambda\end{pmatrix}-\begin{pmatrix}3\\2\\10\end{pmatrix}\right].\begin{pmatrix}-6\\18\\3\end{pmatrix}=0$		M1* Allow one error A1	
					M1* Obtaining a value of λ	*Dep **
			$\lambda=\dfrac{1}{3}$		A1	
			Shortest distance is $\sqrt{(6)^2+(4)^2+(-12)^2}$		M1	*Dep ***
			Shortest distance is 14		A1	

Question			Answer	Marks	Guidance	
1	(iii)		$\begin{pmatrix}11\\0\\-3\end{pmatrix}+\lambda\begin{pmatrix}-6\\18\\k+3\end{pmatrix}=\begin{pmatrix}3\\2\\10\end{pmatrix}+\mu\begin{pmatrix}-1\\4\\1\end{pmatrix}$			
			$11-6\lambda=3-\mu$	M1	Allow one error	Must use different parameters
			$18\lambda=2+4\mu$	A1	Two correct equations	
			$\lambda=5,\quad \mu=22$	A1		
			$-3+\lambda(k+3)=10+\mu$	M1	Obtaining a value of k	*Other methods possible*
			$k=4$	A1		*(e.g. distance between lines is 0)*
			Point of intersection is $\begin{pmatrix}3\\2\\10\end{pmatrix}+22\begin{pmatrix}-1\\4\\1\end{pmatrix}$	M1		
			Point of intersection is $(-19,\ 90,\ 32)$	A1 **[7]**		
1	(iv)		$\left\|\begin{pmatrix}-1\\4\\1\end{pmatrix}\right\|=\sqrt{18}$, so $\overrightarrow{AD}=(\pm)\dfrac{12}{\sqrt{18}}\begin{pmatrix}-1\\4\\1\end{pmatrix}=2\sqrt{2}\begin{pmatrix}-1\\4\\1\end{pmatrix}$	M1* A1	Obtaining $\overrightarrow{AD}$ or D	
			Volume is $\frac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC}).\overrightarrow{AD}$	M1*	Appropriate scalar triple product	
			$=\dfrac{1}{6}\left[\begin{pmatrix}8\\-2\\-13\end{pmatrix}\times\begin{pmatrix}2\\16\\-10\end{pmatrix}\right].(2\sqrt{2})\begin{pmatrix}-1\\4\\1\end{pmatrix}$	A1 ft	Correct expression	*Can be implied*
			$=\dfrac{\sqrt{2}}{3}\begin{pmatrix}228\\54\\132\end{pmatrix}.\begin{pmatrix}-1\\4\\1\end{pmatrix}=\dfrac{\sqrt{2}}{3}(120)$	M1	Evaluating scalar triple product	*Dep* **
			$=40\sqrt{2}$	A1 **[6]**	Accept 56.6	

Question			Answer	Marks	Guidance	
2	(i)		$\frac{\partial z}{\partial x} = 6x^2 + 6x + 12y$	B1		
			$\frac{\partial z}{\partial y} = 6y^2 + 6y + 12x$	B1		
			If $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$, $6x^2 + 6x + 12y = 6y^2 + 6y + 12x$			
			$x^2 - y^2 - x + y = 0$			
			$(x-y)(x+y-1) = 0$	M1	Identifying factor $(x-y)$	*SC* If M0, then give
			$y = x$ or $y = 1-x$	E1E1		B1 for verifying $y = x$ B1 for verifying $y = 1-x$
				[5]		
2	(ii)		$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0$	M1		Can be implied
			If $y = x$ then $6x^2 + 6x + 12x = 0$	M1	Obtaining quadratic in x (or y)	Or quartic, and factorising as x(linear)(quadratic)
			$x = 0, \ -3$	M1	Obtaining a non-zero value of x	
			Stationary points $(0, \ 0, \ 0)$ and $(-3, \ -3, \ 54)$	B1A1	Condone $(0, \ 0)$ for B1	
			If $y = 1-x$ then $6x^2 + 6x + 12(1-x) = 0$			
			$x^2 - x + 2 = 0$	M1	Obtaining quadratic with no real roots	
			Which has no real roots ($D = -7 < 0$)	A1	Correctly shown	Just stating 'No real roots' M1A0
				[7]		
2	(iii)		At P, $\frac{\partial z}{\partial x} = \frac{21}{2}$, $\frac{\partial z}{\partial y} = \frac{21}{2}$	M1 A1	Substituting into $\frac{\partial z}{\partial x}$ or $\frac{\partial z}{\partial y}$	Correct value, or substitution seen
			$\delta z \approx \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y$	M1		
			$w \approx \frac{21}{2}h + \frac{21}{2}h$	A1 ft		
			$h \approx \frac{w}{21}$	A1		

Question			Answer	Marks	Guidance	
				[5]		
2	(iv)		$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=24$	M1	Allow sign error	24λ is M0 unless $\lambda=\pm1$ appears later
			If $y=x$ then $6x^2+6x+12x=24$ $x=1,\ -4$	M1	Obtaining quadratic in x (or y)	Or quartic, and one linear factor
			Points $(1,\ 1,\ 22)$ and $(-4,\ -4,\ 32)$	A1A1	If neither correct, give A1 for $x=1,\ -4$	
			If $y=1-x$ then $6x^2+6x+12(1-x)=24$ $x=2,\ -1$	M1	Obtaining quadratic in x (or y)	Or third linear factor of quartic
			Points $(2,\ -1,\ 5)$ and $(-1,\ 2,\ 5)$	A1A1 [7]	If neither correct, give A1 for $x=2,\ -1$	
3	(a)		$r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2=a^2(1+\cos\theta)^2+(-a\sin\theta)^2$	B1	Condone ... $+(a\sin\theta)^2$	
			$=a^2(1+2\cos\theta+\cos^2\theta+\sin^2\theta)=2a^2(1+\cos\theta)$		or $4a^2\cos^4\frac{1}{2}\theta+4a^2\sin^2\frac{1}{2}\theta\cos^2\frac{1}{2}\theta$	
				M1	Using $1+\cos\theta=2\cos^2\frac{1}{2}\theta$	
			$=4a^2\cos^2\frac{1}{2}\theta$	A1		
			Arc $\int\sqrt{r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta=\int_0^{\frac{1}{2}\pi}2a\cos\frac{1}{2}\theta\,\mathrm{d}\theta$	M1	For $\int\sqrt{r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ in terms of θ	Limits not required
			$=\left[4a\sin\frac{1}{2}\theta\right]_0^{\frac{1}{2}\pi}$	A1	For $4a\sin\frac{1}{2}\theta$	
			$=2\sqrt{2}\,a$	A1 [6]		

Question			Answer	Marks	Guidance	
3	(b)	(i)	$1+\left(\frac{dy}{dx}\right)^2 = 1+\left(\frac{x^2}{2}-\frac{1}{2x^2}\right)^2$	B1		
			$= \frac{x^4}{4}+\frac{1}{2}+\frac{1}{4x^4}$	M1		
			$= \left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2$	A1		
			Area is $\int 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$	M1*		
			$= \int_1^2 2\pi\left(\frac{x^3}{6}+\frac{1}{2x}\right)\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)dx$	A1 ft	Integral expression including limits	
			$= 2\pi\int_1^2\left(\frac{x^5}{12}+\frac{x}{3}+\frac{1}{4x^3}\right)dx$	M1	Obtaining integrable form	*Dep* *
			$= 2\pi\left[\frac{x^6}{72}+\frac{x^2}{6}-\frac{1}{8x^2}\right]_1^2$	A1	Allow one error	
			$= \frac{47\pi}{16}$	A1 **[8]**		

Question			Answer	Marks	Guidance	
3	(b)	(ii)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x + \frac{1}{x^3} \quad (= \frac{17}{8})$	B1		
			$\rho = \frac{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^3}{x + \frac{1}{x^3}}$	M1 A1 ft	Using formula for ρ or κ Correct expression for ρ or κ	
			$= \frac{\left(1 + \left(\frac{15}{8}\right)^2\right)^{\frac{3}{2}}}{2 + \frac{1}{8}} = \frac{\left(\frac{17}{8}\right)^3}{\frac{17}{8}}$	A1 ft	Correct numerical expression for ρ	
			$= \frac{289}{64}$	E1 **[5]**	Correctly shown	
3	(b)	(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{15}{8}$, so unit normal is $\frac{1}{17}\begin{pmatrix} -15 \\ 8 \end{pmatrix}$	M1 A1	Obtaining a normal vector Correct unit normal	Allow M1 for $\begin{pmatrix} \pm 8 \\ \pm 15 \end{pmatrix}$ or $\begin{pmatrix} \pm 15 \\ \pm 8 \end{pmatrix}$
			$\mathbf{c} = \begin{pmatrix} 2 \\ 19/12 \end{pmatrix} + \frac{289}{64}\begin{pmatrix} -15/17 \\ 8/17 \end{pmatrix}$	M1	Allow sign errors	Must use a unit vector
			Centre of curvature is $\left(-\frac{127}{64}, \frac{89}{24}\right)$	A1A1 **[5]**		

Question			Answer	Marks	Guidance	
4	(a)	(i)	Identity is e Element: a b c d e f g h Inverse: b a c g e h d f	B1 B2 **[3]**	Give B1 for four correct	
4	(a)	(ii)	$d^2 = a,\ \ d^4 = c$ Hence d has order 8, and G is cyclic	M1 A1 A1 E1 **[4]**	Finding powers of an element Identifying d (or f or g or h) as a generator Or $f^2 = b,\ \ f^4 = c$ Or $g^2 = b,\ \ g^4 = c$ Or $h^2 = a,\ \ h^4 = c$ Correctly shown	At least fourth power *Implies previous M1*
4	(a)	(iii)	H: 0 2 4 6 8 10 12 14 G: e d a f c h b g or: e f b d c g a h or: e g b h c f a d or: e h a g c d b f	B1 B1 B1 **[3]**	For $e \leftrightarrow 0$ and $c \leftrightarrow 8$ For $\{d, f, g, h\} \leftrightarrow \{2, 6, 10, 14\}$ For a fully correct isomorphism	In any order
4	(a)	(iv)	Rotations have order 2 or 4 Reflections have order 2 There is no element of order 8 Hence not isomorphic	B1 E1 **[2]**	Correct statement about rotations and/or reflections which implies non-IM *Or* More than one element of order 2 *Or* Not commutative Fully correct explanation	*Or* (4) reflections (and 180° rotation) have order 2 *Or* composition of reflections (or 90° rotation and reflection) is not commutative Dependent on previous B1

Question			Answer	Marks	Guidance	
4	(b)	(i)	$\mathrm{f}_m\mathrm{f}_n(x) = \dfrac{\dfrac{x}{1+nx}}{1+m\left(\dfrac{x}{1+nx}\right)}$	M1	Composition of functions	In either order
			$= \dfrac{x}{1+nx+mx} = \dfrac{x}{1+(m+n)x} = \mathrm{f}_{m+n}(x)$	E1	Correctly shown	E0 if in wrong order
				[2]		
4	(b)	(ii)	$(\mathrm{f}_m\mathrm{f}_n)\mathrm{f}_p = \mathrm{f}_{m+n}\mathrm{f}_p = \mathrm{f}_{m+n+p}$ $\mathrm{f}_m(\mathrm{f}_n\mathrm{f}_p) = \mathrm{f}_m\mathrm{f}_{n+p} = \mathrm{f}_{m+n+p}$	M1	Combining three functions	
			Hence S is associative	E1	Correctly shown	M1E1 bod for $(\mathrm{f}_m\mathrm{f}_n)\mathrm{f}_p = \mathrm{f}_{m+n+p} = \mathrm{f}_m(\mathrm{f}_n\mathrm{f}_p)$
				[2]		
4	(b)	(iii)	For any f_m, f_n in S, $\mathrm{f}_m\mathrm{f}_n = \mathrm{f}_{m+n}$	M1	Referring to this in context	
			$\mathrm{f}_m\mathrm{f}_n$ is in S (so S is closed)	A1		
			Identity is f_0	B1	B0 for x B1 for $n=0$	
			Inverse of f_n is f_{-n}	B1		
			since $\mathrm{f}_n\mathrm{f}_{-n} = \mathrm{f}_{n-n} = \mathrm{f}_0$	B1		
			S is also associative, and hence is a group	E1	Closure, associativity, identity and inverses must all be mentioned in (iii)	Dependent on previous 5 marks
				[6]		
4	(b)	(iv)	$\{\mathrm{f}_{2n}\}$ for all integers n	B2	Or $\{\mathrm{f}_{3n}\}$, etc Give B1 for multiples of 2 (or 3, etc) but not completely correctly described	e.g. $\{\mathrm{f}_0, \mathrm{f}_2, \mathrm{f}_4, \mathrm{f}_6, \ldots\}$
				[2]		

Question			Answer	Marks	Guidance	
5			***Pre-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	
	(i)		$\mathbf{P} = \begin{pmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0.05 & 0.5 & 0 & 0 \\ 0 & 0.45 & 0.05 & 0.5 & 0 \\ 0 & 0 & 0.45 & 0.05 & 0 \\ 0 & 0 & 0 & 0.45 & 1 \end{pmatrix}$	B3 **[3]**	Give B2 for four columns correct Give B1 for two columns correct	
5	(ii)		$\mathbf{P}^8 \begin{pmatrix} 0 \\ 1/3 \\ 1/3 \\ 1/3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5042 \\ 0.0230 \\ 0.0278 \\ \mathbf{0.02071} \\ 0.4242 \end{pmatrix}$ P(3 lives) = 0.0207 (4 dp)	M1 E1 **[2]**	For $\mathbf{P}^8$ (allow $\mathbf{P}^7$ or $\mathbf{P}^9$) and initial column matrix Correctly shown	
5	(iii)		Let q(n) = P(not yet ended after n tasks) $= \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \end{pmatrix} \mathbf{P}^n \begin{pmatrix} 0 \\ 1/3 \\ 1/3 \\ 1/3 \\ 0 \end{pmatrix}$ $\text{q}(10) = 0.0371$	M1 M1 A1 **[3]**	Obtaining probabilities after 10 tasks Adding probabilities of 1, 2, 3 lives	Allow M1 for using $\mathbf{P}^9$ or $\mathbf{P}^{11}$

Question			Answer	Marks	Guidance	
5	(iv)		$q(9) - q(10)$ $= 0.05072 - 0.03709$ $= 0.0136$	M1 M1 A1 **[3]**	Using $q(9)$ and $q(10)$ Evaluating $q(9)$	
		OR	$\mathbf{P}^9 \begin{pmatrix} 0 \\ 1/3 \\ 1/3 \\ 1/3 \\ 0 \end{pmatrix} = \begin{pmatrix} . \\ 0.01506 \\ . \\ 0.01355 \\ . \end{pmatrix}$ $0.01506 \times 0.5 + 0.01355 \times 0.45$ $= 0.0136$		M1 Probs of 1 and 3 lives after 9 tasks M1 A1	
5	(v)		$q(13) = 0.01374$ $q(14) = 0.00998$ Smallest N is 14	M1 M1 A1 **[3]**	Evaluating $q(n)$ for some $n > 10$ Consecutive values each side of 0.01 Must be clear that their answer is 14	Just $N = 14$ www earns B3
5	(vi)		$\mathbf{P}^n \to \begin{pmatrix} 1 & 0.7880 & 0.5525 & 0.2908 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2120 & 0.4475 & 0.7092 & 1 \end{pmatrix} = \mathbf{L}$	B2 **[2]**	Give B1 for any element correct to 3 dp (other than 0 or 1)	
5	(vii)		$\mathbf{L} \begin{pmatrix} 0 \\ 1/3 \\ 1/3 \\ 1/3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5438 \\ 0 \\ 0 \\ 0 \\ 0.4562 \end{pmatrix}$ P(wins a prize) = 0.4562	M1M1 A1 **[3]**	Using $\mathbf{L}$ and the initial column matrix	

Question			Answer	Marks	Guidance	
5	(viii)		Maximum probability is 0.7092 Always start with 3 lives	B1 ft B1 **[2]**		
5	(ix)		$\mathbf{L}\begin{pmatrix}0\\0.1\\p\\q\\0\end{pmatrix}=\begin{pmatrix}0.4\\0\\0\\0\\0.6\end{pmatrix}$ $0.7880\times0.1+0.5525p+0.2908(0.9-p)=0.4$ P(2 lives) = 0.2273 , P(3 lives) = 0.6727	M1 M1 A1 **[3]**	Or $0.0212+0.4475p+0.7092(0.9-p)=0.6$ Obtaining a value for p or q Accept values rounding to 0.227, 0.673	Allow use of $p+q=1$
5			***Post-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	
5	(i)		$\mathbf{P}=\begin{pmatrix}1&0&0&0&0\\0.5&0.05&0.45&0&0\\0&0.5&0.05&0.45&0\\0&0&0.5&0.05&0.45\\0&0&0&0&1\end{pmatrix}$	B3 **[3]**	Give B2 for four rows correct Give B1 for two rows correct	
5	(ii)		$\begin{pmatrix}0&\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&0\end{pmatrix}\mathbf{P}^8$ $=\begin{pmatrix}0.5042&0.0230&0.0278&\mathbf{0.02071}&0.4242\end{pmatrix}$ P(3 lives) = 0.0207 (4 dp)	M1 E1 **[2]**	For $\mathbf{P}^8$ (allow $\mathbf{P}^7$ or $\mathbf{P}^9$) and initial row matrix Correctly shown	

Question			Answer	Marks	Guidance	
5	(iii)		Let q(n) = P(not yet ended after n tasks) $= \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} \mathbf{P}^n \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$	M1 M1	Obtaining probabilities after 10 tasks Adding probabilities of 1, 2, 3 lives	Allow M1 for using $\mathbf{P}^9$ or $\mathbf{P}^{11}$
			$\text{q}(10) = 0.0371$	A1 **[3]**		
5	(iv)		$\text{q}(9) - \text{q}(10)$ $= 0.05072 - 0.03709$ $= 0.0136$	M1 M1 A1 **[3]**	Using q(9) and q(10) Evaluating q(9)	
		OR	$\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} \mathbf{P}^9$ $= \begin{pmatrix} . & 0.01506 & . & 0.01355 & . \end{pmatrix}$ $0.01506 \times 0.5 + 0.01355 \times 0.45$ $= 0.0136$		M1 Probs of 1 and 3 lives after 9 tasks M1 A1	
5	(v)		$\text{q}(13) = 0.01374$ $\text{q}(14) = 0.00998$ Smallest N is 14	M1 M1 A1 **[3]**	Evaluating q(n) for some $n > 10$ Consecutive values each side of 0.01 Must be clear that their answer is 14	Just $N = 14$ www earns B3
5	(vi)		$\mathbf{P}^n \to \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.7880 & 0 & 0 & 0 & 0.2120 \\ 0.5525 & 0 & 0 & 0 & 0.4475 \\ 0.2908 & 0 & 0 & 0 & 0.7092 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \mathbf{L}$	B2 **[2]**	Give B1 for any element correct to 3 dp (other than 0 or 1)	

Question			Answer	Marks	Guidance	
5	(vii)		$\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}\mathbf{L}$ $= \begin{pmatrix} 0.5438 & 0 & 0 & 0 & 0.4562 \end{pmatrix}$ P(wins a prize) = 0.4562	M1M1 A1 **[3]**	Using **L** and the initial row matrix	
5	(viii)		Maximum probability is 0.7092 Always start with 3 lives	B1 ft B1 **[2]**		
5	(ix)		$\begin{pmatrix} 0 & 0.1 & p & q & 0 \end{pmatrix}\mathbf{L}$ $= \begin{pmatrix} 0.4 & 0 & 0 & 0 & 0.6 \end{pmatrix}$ $0.7880 \times 0.1 + 0.5525p + 0.2908(0.9 - p) = 0.4$ P(2 lives) = 0.2273 , P(3 lives) = 0.6727	 M1 M1 A1 **[3]**	Or $0.0212 + 0.4475p + 0.7092(0.9 - p) = 0.6$ Accept values rounding to 0.227, 0.673	Allow use of $p + q = 1$